

Geometry may be regarded as true; from this fact can be deduced a group of equations typified by

$$\frac{da}{da} = \frac{f(b)}{\sin \gamma}, \quad \frac{db}{da} = \cos \gamma, \quad f'(b) = -\frac{d\gamma}{da},$$

where  $a, b, c$ , are the sides of a triangle, and  $\alpha, \beta, \gamma$ , the corresponding angles. From these, by appropriate eliminations and transformations, the differential equation

$$\{f(a)\}^2 = -\kappa^2[1 - \{f'(a)\}^2]$$

can be found for the function  $f$ . Solving this, we have

$$f(a) = \kappa \sinh \frac{a}{\kappa},$$

and thence can derive the fundamental equations of non-Euclidian measurement.

$$\sinh \frac{a}{\kappa} / \sinh \alpha = \sinh \frac{b}{\kappa} / \sin \beta = \sinh \frac{c}{\kappa} / \sin \gamma.$$

This was followed by the communication of a Report on the Problem of Three Bodies, which Mr. E. T. Whittaker was commissioned to prepare at the Toronto meeting. In a general sketch of the results, Mr. Whittaker explained the transformation which has taken place in dynamical astronomy as a result of the researches of Newcomb, Hill, Lindstedt and Poincaré. Formerly the subject might be said to consist of two departments—the planetary and lunar theories; now the distinction between these was becoming less prominent, as the Problem of Three Bodies was treated in greater generality. Among the advances referred to were Dr. Hill's introduction of periodic orbits as a substitute for Keplerian ellipses in the first approximation to the solution, Newcomb's proof that the problem can be solved by series in which the time occurs only in the arguments of trigonometric functions, Poincaré's theorem that these series are only asymptotic expansions, and Bruns' result that the system possesses no algebraic integrals other than those already known.

A second paper by Prof. Forsyth, "On Singular Solutions of Ordinary Differential Equations," described some properties of the  $p$ -discriminant and  $c$ -discriminant of an ordinary differential equation of the first order. The two last papers on the list were "An Application and Interpretation of Infinitesimal Transformations," by Dr. E. O. Lovett, of Princeton University, N.J.; and "On Fermat's Numbers," by Lieut.-Colonel Cunningham. In the absence of their authors the papers were communicated by title, and the session was closed.

Looking at the papers as a whole, they were of just that character which makes the B.A. meeting useful to mathematicians; that is, they related not so much to abstruse continuations of well-known theories as to new and little-known subjects, suggestions of improved notations, reports on the recent progress of different branches of mathematics, and generally all those topics for which discussion at a real meeting is more important than the publication of a paper.

#### PHYSICS AT THE BRITISH ASSOCIATION.

THE attendance of physicists at Dover was rather smaller than usual, on account of the occurrence of the Volta Centenary celebrations at Como and the simultaneous meetings of the French Association for the Advancement of Science at Boulogne. Several of those who in past years have been leaders in the discussions of Section A were this year conspicuous by their absence. Nevertheless, the papers read maintained a high standard of excellence, and the reports presented indicate that good work is being done by the committees appointed for scientific research.

The address delivered by Prof. Poynting, as President of the Section, was the subject of many conversations, not only among physicists but with biologists also; the existence of the sharp line which he indicated between the psychical and physical methods and the phenomena to which each is applicable, was acknowledged on all sides. The physicists were divided on the question of the danger of too much hypothesis, and especially on the possibility of the propagation of electromagnetic waves in air being due to the air as much as to the ether. All, however, were agreed in the expression of thanks to the President, proposed by Sir George Stokes and seconded by Sir Norman Lockyer.

In a paper on the spectroscopic examination of contrast phenomena, Mr. G. J. Burch described experiments which lend

great support to the Young-Helmholtz theory of colour-vision. If the eye is fatigued by exposure to a very intense red light, such as sunlight filtered through red screens and focussed on the eye, and a spectrum be then looked at, the red is invisible; but the rest of the spectrum, green to violet, appears in its ordinary colours. Red-blindness is therefore not accompanied by green-blindness, as Hering's theory requires. Further experiments on the blue and violet portions of the spectrum have led Mr. Burch to the conclusion that we have separate primary sensations for blue and violet, in addition to those for red and green, making four altogether instead of the three postulated by the Young-Helmholtz theory. The experiments are the more convincing because carried out with spectral colours, thus avoiding all errors due to the impurity of pigment colours. In the discussion on the paper several members took part; Sir George Stokes said experiments led him to believe that lobelia blue is a primary sensation, and Principal Glazebrook suggested that the theory should be tested by colour-matches on a spectrophotometer.

Prof. Callendar gave the preliminary results of a research on the variation of the specific heat of water with temperature, which he commenced in Montreal with Mr. H. T. Barnes, and which is now being continued by the latter. The method of experiment consists in allowing water to flow steadily through a narrow tube along which a platinum wire runs axially; on passing a constant electric current through the wire the water finally acquires a steady temperature-difference between the inlet and outlet of the tube, which is measured by platinum thermometers and automatically recorded. Radiation corrections are reduced to a minimum by surrounding the tube with a vacuum-jacket, and the electrical energy supplied is measured by observing the current and the potential-difference between the ends of the wire in the tube. The results show that the specific heat of water has a minimum value of 0.995 in the neighbourhood of 40°C., it rises to 1.000 as the temperature falls to 10°C., and continues to rise rapidly as the temperature decreases. On increasing the temperature above 40°C. the specific heat rises to 0.997 at 60°C. Further experiments will be made in the neighbourhood of the freezing point and on either side of it.

The committee on electrolysis and electro-chemistry has undertaken the comparison of the variation of electrical conductivity with concentration, and the variation of freezing point with concentration for identical very dilute aqueous solutions of electrolytes. The electrical measurements have been successfully carried out by Mr. W. C. Whetham, but the freezing point determinations, undertaken by Mr. E. H. Griffiths, have been delayed by the discovery of errors arising from the presence of dissolved gases in the solutions. Incidentally Mr. Griffiths remarked that he was able to measure temperatures to within three or four parts in a million.

Dr. R. A. Lehfeldt, at a subsequent meeting, called attention to a flaw in Nernst's theory of electrolytic solution pressure. According to this theory, when a metal is immersed in an electrolyte ions are torn either from the metal or from the solution according as the solution-pressure is greater or less than the osmotic pressure of the ions in solution. It is usually supposed that the mass of the ions deposited or dissolved is so extremely small that it cannot be detected; the author showed, however, by considering the electrostatic tension due to the ionic charges, that the amount dissolved should be easily weighable, at any rate in the case of zinc.

The stability of an ether containing long, thin, empty vortex filaments was discussed in a communication by Prof. Fitzgerald on the energy per cubic centimetre in a turbulent liquid transmitting laminar waves. Lord Kelvin considered this subject in 1887, and concluded that rapid diffusion would make the structure unstable. The author held the opinion (though possibly Lord Kelvin would differ from him) that the turbulency of a sufficiently fine-grained irregularly turbulent liquid would ultimately diffuse so slowly that Lord Kelvin's investigation could be applied to it.

Until the meeting of the Association in 1893, it was generally supposed that the absence of an atmosphere from the moon, and of hydrogen from our own atmosphere, is due to the high average velocity of the gaseous molecules, which is sufficient to carry them beyond the range of the moon's or earth's attraction. On that occasion Prof. Bryan demonstrated the incorrectness of this view for the case of the moon, and he has since extended his calculations to the cases of hydrogen and helium in the

earth's atmosphere, and of water vapour in the atmosphere of Mars. The method of calculation is to determine the number of years which would be required for the planet to lose from its surface a layer of the gas one centimetre thick at various temperatures. The results show that the earth might retain helium, but would lose hydrogen appreciably at ordinary temperatures, and that Mars might retain water vapour at ordinary temperatures. If helium ever existed on the earth's surface, it must have escaped when the surface was much hotter than at present, whereas a smaller elevation of temperature would cause water vapour to escape from the surface of Mars.

Prof. W. F. Barrett described the thermo-electric properties of an alloy containing iron 68.8 per cent., nickel 25.0, manganese 5.0, and carbon 1.2. When a thermo-electric couple is formed of this metal and iron, the electromotive force rises with temperature to 300° C.; it then remains steady until 500° C. is reached, after which it falls slightly and rises again to 1100° C.; the fluctuations of electromotive force do not exceed 4 per cent. of the total value. When the alloy forms a couple with nickel the results are similar, but the range of variation is slightly greater.

The committee on the heat of combination of metals in the formation of alloys, appointed last year to assist Dr. A. Galt in his experiments on this subject, reported the completion of their work. Only alloys of zinc and copper have been examined, twenty-two in number and containing from 5 to 90 per cent. of copper; the difference between the amounts of heat evolved by dissolving in nitric acid unit mass of the alloy and corresponding amounts of the mixed metals was taken as the heat of combination of the metals. The results indicate a negative heat of combination for alloys rich in zinc, the numerical value of which is a maximum when the alloy contains 16 per cent. of copper. The formation of an alloy containing about 24 per cent. of copper takes place without absorption or evolution of heat, while for 38 per cent. of copper the heat of combination is a maximum and positive; beyond this it diminishes to zero for pure copper. In the absence of Dr. Galt and other members of the committee no reply was given to a serious criticism by Prof. Vernon Harcourt, that in the experiments no account was apparently taken of the fact that the products arising from the solution of an alloy in nitric acid are not the same as would be obtained from the mixed metals. In his paper read last year at Bristol, Dr. Galt mentioned that he had made many preliminary experiments, and possibly he has examined this point; if not, the results obtained by the committee will be somewhat vitiated.

A preliminary report of the committee on radiation from a source of light in a magnetic field was communicated to the Section, the chief points in which were (1) the discovery that light passing through a magnetic field at right angles to the lines of force suffers absorption (see *NATURE*, vol. lix. pp. 228-9, January 5, 1899); (2) the various modified forms of triplet are true magnetic perturbations of the same kind as the normal triplet; (3) the spectral lines of a substance may be divided into groups such that all members of one group suffer the same kind of perturbation (see *NATURE*, vol. lix. p. 248, January 12, 1899).

The Zeeman effect is attributed to the action of a magnetic field on the moving ions; recently Mr. C. E. S. Phillips has discovered an apparently cognate phenomenon, which he described in his paper on the production in rarefied gases of luminous rings in rotation about lines of magnetic force. An electric discharge is passed between soft iron electrodes in a Crookes' vacuum tube; on stopping the discharge and setting up a magnetic field between the electrodes, a luminous ring forms with its plane at right angles to the lines of force and in rotation about the magnetic axis. The direction of rotation is that which would be communicated to negatively charged particles, and is reversed on reversing the magnetic field; the luminosity persists sometimes for a minute, and reversal of the magnetic field causes it to brighten momentarily. Two explanations of the phenomenon have been given; one is that the rotating matter consists of ions or electrons, and the other that the matter consists of gas particles which have acquired a negative charge by contact with the walls of the tube. From experiments of Prof. J. J. Thomson, it appears that negative ions move more quickly than positive, which would account for the greater luminosity of the negative ions when set in rotation.

In a note on deep-sea waves, Mr. V. Cornish endeavoured to trace relations between the amplitude, wave-length, and wind-

velocity for waves on the surface of deep water. Sir George Stokes pointed out that the amplitude observed is not that of a simple wave, but is the resultant effect of a train of waves of different periods and lengths.

At the meeting of the Section on Saturday the visitors from the French Association at Boulogne were present, and the President extended to them a hearty welcome, which was acknowledged by M. Benoit, as president of the Physical Section of the French Association. A paper was then communicated by Prof. J. J. Thomson, on the existence of masses smaller than the atoms. He stated that several lines of research lead to a determination of the ratio of the mass of an atom ( $m$ ) to the charge carried by the atom ( $e$ ). Among these are electrolysis, the velocity of charged particles in a magnetic field, and the magnetic deflexion of cathode rays. The two latter methods are comparatively simple, because they depend on the observation of luminous effects, but although they agree with each other fairly well, they furnish a value of  $m/e$  which is about 1/1000 of that calculated from electrolytic phenomena. It becomes, therefore, a matter for inquiry whether in the former experiments the atom carries a charge greater than that required by Faraday's laws, or whether the charge is carried by a portion only of the atom—in other words, whether a small fraction of the mass of the atom is detachable which has associated with it a negative charge. The simplest crucial experiment is obtained by determining separately either  $m$  or  $e$ , and the author has devised a means of measuring the latter quantity. He takes a negatively charged metal plate supported horizontally; below this and parallel to it is a very large perforated metal plate, the whole being in rarefied gas at a pressure of about 1/100 mm. mercury. When ultra-violet radiation is directed through the perforated plate to strike the upper plate the latter is discharged, the discharging particles moving along straight lines normal to the two plates. If a magnetic field be now excited with its lines of force parallel to the plates, the particles describe curved paths which are in fact portions of cycloids. When the plates are near together the particles which leave the upper one strike the lower one; if, however, the plates are separated further, the vertex of the cycloidal path comes between them, and the particles do not reach the lower plate, so that the discharge ceases. In the actual experiment there is a gradual, but not abrupt, change in the rate of discharge, possibly because all the particles do not start from the surface of the upper plate. From observations on the distance apart of the plates when the change in the rate of discharge commences, the form of the cycloidal path is determined, and the results show that the smaller value of  $m/e$  is applicable to this case and to that of illumination by cathode rays. Further, the amount of electricity discharged by the illuminated plate per second is proportional to the number of particles between the plates, to the charge carried by each ( $e$ ), and to the velocity of the particles. The last-named quantity is measured by a method due to Prof. Rutherford, so that if the total number of particles in the space is known the value of  $e$  can be determined. To count the particles use is made of the fact that they serve as nuclei for the formation of drops out of a condensing vapour, each particle giving rise to one drop. Let a known amount of air of given humidity be suddenly and definitely expanded in the presence of the particles, and observe the rate at which the drops fall; this rate gives the size of the drops, and hence their mass, and since the whole mass of water deposited is known, the number of drops is thus determined. For negative charges the ratio  $m/e$  is independent of the nature of the gas, whereas for positive charges its value varies from one gas to another, and corresponds generally with the values given by electrolytic phenomena. Prof. Thomson considers that electrification consists in the removal from the "atom" of a small corpuscle with which the negative charge is associated; the remaining large portion of the mass is positively charged. This view is supported by Prout's hypothesis that the mass of an atom is not invariable, and by the evidence derived by Lockyer and others from spectroscopic observations.

In the discussion which followed upon Prof. Thomson's paper, M. Broca described spectroscopic observations of a spark obtained between two platinum electrodes  $\frac{1}{2}$  mm. apart in a Crookes' vacuum tube; the spectra of the regions near the electrodes and the space between them were not alike. Prof. Rücker drew attention to Schuster's experiments, in which the spectrum of a substance not present in the material examined sprang into being in the arc itself. He believed matter to be a



complicated collection of units themselves similar. Sir Norman Lockyer said that if we accept the view that elements of smallest atomic weights should appear first in the spectrum of a hot star, we must assume the existence of forms of calcium, magnesium, iron and copper having atomic weights which are submultiples of those assigned to them in ordinary chemistry. Further, the division of the spectra of certain elements into series of lines by Rydberg, Runge and Paschen, and others indicates that the atoms of these elements are complexes; we have, therefore, no reason to suppose that the so-called "atoms" are not dissociable at high temperatures. Prof. Oliver Lodge thought the investigations of Prof. Thomson might turn out to be the discovery of an electric inertia, and lead to a theory of mass. Several speakers expressed their pleasure in receiving the members of the French Association.

In the very short time remaining after the discussion on the previous paper, Prof. Oliver Lodge gave a short account of the controversy respecting the seat of Volta's contact force.

On Monday the Section was subdivided for papers on mathematics and meteorology respectively. In the latter department, over which Sir George Stokes presided, a formal report was presented by the committee on solar radiation. Dr. van Rijckevorsel read a paper in which he traced an intimate connection between the activity of sun-spots and the temperature. The committee on seismology presented a voluminous report on their work, from which it appears that twenty-three stations are now equipped with recording seismographs, and registers have been received from ten of these. Notes on these registers occupy a considerable portion of the report; the rest of the report is abstracted from articles which have already appeared in *NATURE* (February 16 and March 1, 1899). Mr. T. F. Claxton communicated the preliminary results of a year's work with the seismograph at Mauritius. The diurnal waves are of greater amplitude than at any other observing station, and there is a well-marked bi-diurnal effect possibly connected with barometric pressure. Rapid and large changes of the vertical have occurred on several occasions, in addition to a constant gradual change. Air tremors have given trouble at night. The earthquake effects have been of disappointingly small amplitude, and it is suggested that the ocean may act as a damper to earthquake shocks.

Mr. A. L. Rotch gave an interesting account of the progress achieved during the past year at Blue Hill, Massachusetts, in the exploration of the air with kites. The Hargrave kite with curved surfaces has been found more satisfactory than any other form, and the meteorograph records temperature, humidity, height and wind. Temperature is found to decrease at first with elevation, and afterwards to increase again. The heights attained were on the average greater than in previous years. The author mentioned that the United States Government has arranged for daily simultaneous observations at two heights in the case of a number of stations, the kite being used for the high-level observations. The results are not quite satisfactory, because kites could not be sent up on some days; it is suggested that on such occasions a captive balloon be employed. Prof. Thomson hoped that the variation of atmospheric electric potential would be investigated by means of kites. Prof. G. H. Darwin regretted that on account of the non-existence of a Government meteorological observatory, this country is very backward in the adoption of recent methods of meteorological research. In a subsequent paper Mr. Rotch gave an account of the first crossing the Channel by a balloon, by Dr. Jeffries and M. Blanchard in January 1785. The former was a Harvard graduate in medicine, who settled in London, and the latter a French professional aeronaut. The expedition was of a scientific character.

A description of the hydro-aërograph, an apparatus invented by Mr. F. Napier Denison for registering small fluctuations of level of the American lakes and simultaneous small changes of air-pressure, was read by Mr. W. N. Shaw. The apparatus is designed to study more minutely an observed effect of barometric changes on the surfaces of the great American lakes.

The Ben Nevis committee presented the usual summary of their records, and stated that the conclusions arrived at last year with reference to the effects of approaching cyclones and anti-cyclones on the two observatories are supported by the examination of later records. The committee on meteorological photography reported having obtained photographs of some rare forms of cloud and some studies of lightning flashes; the structure of thunderclouds appears to resemble two parallel discs of cloud, with lightning flashes passing between them or from one face to the other of either cloud.

On Tuesday, Prof. Threlfall described a portable gravity balance, designed by Prof. Pollock and himself, for the measurement of small differences in the intensity of gravity from place to place. It consists of a light wire attached near one end to the centre of a horizontally stretched and twisted quartz fibre, the moment of the weight of the wire just balancing the torsional moment of the fibre. The wire is only just in stable equilibrium, and the torsion of the fibre is noted when the wire is adjusted to coincide with the axis of a microscope carried on the frame of the apparatus. The instrument can now be relied upon to 1 part in 500,000, but the accuracy of single readings is greater than this. It has been severely tested by much travelling on the Australian coast.

The committee on electric standards reported that Profs. Ayerton and J. V. Jones have now completed the plans and specifications for the ampere balance to be used in constructing an ampere standard. The committee will consider the proposals of Prof. Callendar for the construction of a standard platinum thermometer in terms of which all other platinum thermometers can be compared. The report contains the results of a determination of the coefficient of expansion of porcelain, by Mr. T. G. Bedford, which was undertaken in order to compare the scales of temperature and platinum thermometers of air.

Prof. Callendar opened a discussion on platinum thermometry, in which he advocated the adoption of the variation of resistance of platinum as a basis for a *practical* scale of temperature. He suggested the construction of a standard thermometer from a particular sample of platinum wire, and the use of a parabolic difference formula for the determination of temperature by its means. The difference-coefficient may be obtained by using as a secondary fixed point the boiling point of sulphur (444.53° C. at normal pressure). Dr. J. A. Harker described the method used, and Dr. Chappuis the results obtained, in a comparison of platinum and nitrogen thermometers at the International Bureau of Weights and Measures at Sèvres. The results agree fairly well with those of Callendar and Griffiths in the comparison of the air and platinum thermometers. In the discussion Mr. E. H. Griffiths advocated the use of the platinum thermometer on the ground that only three readings are necessary in order to standardise any instrument. Prof. Carey Foster was of opinion that the electrical method would furnish a good intermediate standard; for absolute values, however, the gas thermometer must be used, because there is no theory of the variation of electrical resistance with temperature and only an empirical knowledge of it. Prof. Burstall described experiments supporting the proposals of Prof. Callendar. Principal Glazebrook thought that, before taking platinum as a standard, experiments should be undertaken to ascertain whether it is superior to other metals, for instance gold. Dr. Chree said that some platinum thermometers purchased by the Kew Observatory had exhibited curious tricks, and were far from satisfactory, because the reasons for departure from accuracy were numerous and not always discoverable. In the case of mercury thermometers the zero certainly alters, but the change has a known cause, and can be allowed for. Prof. Threlfall remarked that for rapid and accurate work the platinum thermometer alone could be used; the enormous heat-capacity of a mercury thermometer rendering it quite unserviceable. Mr. W. N. Shaw thought the thermo-electric couple methods, upon which the Germans are concentrating their attention, ought to be compared with the platinum thermometer before deciding upon a standard. In reply, Prof. Callendar said that methods based on the use of a thermo-electric couple are not sensitive at low temperatures.

On Wednesday, Dr. L. A. Bauer described the arrangements made by the United States Coast and Geodetic Survey for the proposed magnetic survey of the United States and Alaska, and expressed a hope that the Canadian Government would consider the possibility of a simultaneous survey of Canada. Dr. Bauer also described the results of a magnetic survey of Maryland. Dr. E. P. Lewis, in a paper on the spectral sensitiveness of mercury vapour in an atmosphere of hydrogen, described the appearance and intensity of the spectrum of a mixture of hydrogen and vapour of mercury in varying proportions. Mr. J. Gifford, who has measured the angles of prisms of quartz and calcite, and the corresponding minimum deviations for the mean of the sodium lines, at various temperatures, gave an account of the variation of refractive index with temperature in these cases.

The proceedings of the Section closed with votes of thanks to the president and secretaries, proposed by Prof. Forsyth, and seconded by Prof. Reinold.